Longitudinal Magneto-Transport from Anomalous Hydrodynamics

based on [PRD 108 (2023), 1] and [2309.xxxxx] with A. Amoretti, D.K. Brattan and I. Matthaiakakis

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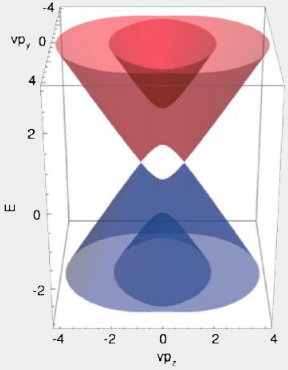


Figure: Typical band structure of WSMs [Armitage *et al.* Rev.Mod.Phys. **90** (2018)]. Weyl nodes have opposite chirality. Examples: NbP, TaAs, TaP, NbAs, WP_2.

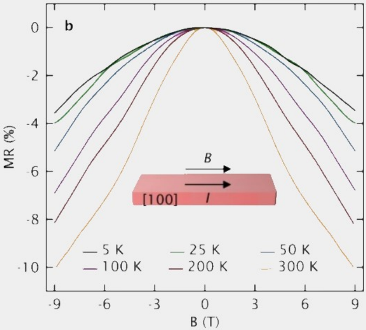


Figure: [Niemann *et al.* Sci.Rep. **7** (2017)]. Conductivity $\sigma \propto B^2$ in NbP.

Relativistic hydrodynamics

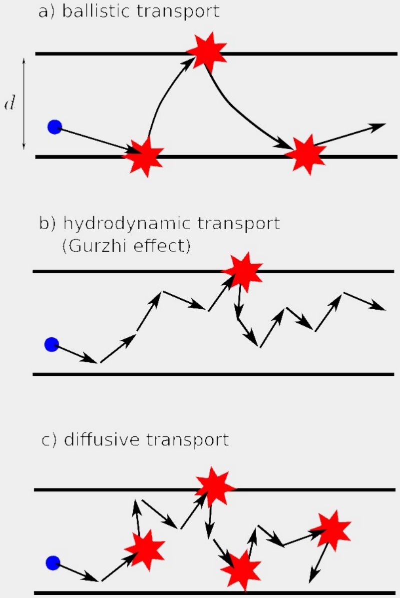
WSMs can have hydrodynamic behaviour [Gooth *et al.* Nat.Comm. **9** (2018)]: $l_{ee} \ll l_{ep}, l_{ei}$.

Conservation of energy, charge and momentum

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu J^\mu = 0$$

Based on:

- derivative counting
- symmetries
- 2nd law of thermodynamics



Anomalous chiral hydrodynamics

Hydrodynamics + chiral anomaly [Son, Surókwa PRL **103** (2009)].

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \qquad \partial_\mu J^\mu = cE \cdot B$$

At order one, with $B \sim \mathcal{O}(\partial)$:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P\Delta^{\mu\nu} + \xi^\epsilon \left(u^\mu B^\nu + u^\nu B^\mu\right) - \zeta\theta\Delta^{\mu\nu} - \eta\sigma^{\mu\nu} + \mathcal{O}(\partial^2)$$
$$J^\mu = nu^\mu + \xi B^\mu + \sigma\left(E^\mu - T\Delta^{\mu\nu}\partial_\nu \frac{\mu}{T}\right) + \mathcal{O}(\partial^2)$$

Chiral Magnetic Effect, Chiral Vortical Effect, …

Transport coefficients ξ, ξ^ϵ fixed by hydrodynamics.

Non-anomalous conductivities

Conductivity from Linear Response Theory

$$M_{ab} \cdot \varphi_b = \lambda_a \qquad \varphi_a = \left(\delta T, \delta\mu, \delta \mathbf{v}^i\right)$$

$$\delta \mathbf{J} = \sigma \delta \mathbf{E}$$

If $J^\mu \sim \mathcal{O}(\partial)$, then numerator of $\sigma \sim \mathcal{O}(\partial)$ must be truncated at order one

$\Rightarrow$ If $B \sim \mathcal{O}(\partial)$, σ cannot depend on $B^2 \sim \mathcal{O}(\partial^2)$.

Standard order-one anomalous hydrodynamics fails to predict negative magnetoresistance — cfr. [Landsteiner *et al.* JHEP **03** (2014), Lucas *et al.* PNAS **113** (2016)]

Solution: consider $B \sim \mathcal{O}(1) \Rightarrow$ now $B^2 \sim \mathcal{O}(1)$ and appears in the conductivity.

Anomalous ideal fluid, $\xi^{\epsilon} = \frac{1}{2}c\mu^2$ and $\xi = c\mu$ [Ammon *et al.* JHEP **04** (2021)].

Conductivity is now *well-defined* and *physical*

$$\sigma(\omega) = \frac{i}{\omega}\left[\frac{n^2}{(P+\epsilon)} + \Xi B^2\right]$$

with

$$\Xi = \frac{c^2 s^2 T^2 \left(\frac{\partial n}{\partial T}\mu - \frac{\partial \epsilon}{\partial T}\right)}{(P+\epsilon)\left(\frac{\partial \epsilon}{\partial \mu}\frac{\partial n}{\partial T} - \frac{\partial \epsilon}{\partial T}\frac{\partial n}{\partial \mu}\right) + B^2 c^2 \mu^2 \left(\frac{\partial \epsilon}{\partial T} - \frac{\partial n}{\partial T}\mu\right)}$$

Problem with DC conductivity

Problem: $\sigma_{\text{DC}} = \lim_{\omega\to 0}\sigma(\omega)$ is *infinite* $\Rightarrow$ need to add charge, energy and momentum relaxations.

Usually [Landsteiner *et al.* JHEP **03** (2014), Abbasi *et al.* JHEP **05** (2019)]

$$\partial_t \delta T^{00} + \ldots = -\frac{\delta T^{00}}{\tau_\epsilon}$$

$$\partial_t \delta T^{0i} + \ldots = -\frac{\delta T^{0i}}{\tau_m}$$

$$\partial_t \delta J^{0} + \ldots = -\frac{\delta J^{0}}{\tau_n}$$

Onsager relations: $\tau_m = \tau_\epsilon = \tau_n \Rightarrow$ unphysical choice

Generalized relaxations

Add all relaxations allowed by symmetry:

$$\partial_t \delta\epsilon + \ldots = -\frac{\delta\epsilon}{\tau_{\epsilon\epsilon}} - \frac{\delta n}{\tau_{\epsilon n}}$$
$$\partial_t \delta n + \ldots = -\frac{\delta\epsilon}{\tau_{n\epsilon}} - \frac{\delta n}{\tau_{nn}}$$
$$\partial_t \delta v^i + \ldots = -\frac{\delta v^i}{\tau_m}$$

Onsager relations:

$$\tau_m \geq 0 \qquad\qquad \frac{\chi_{\epsilon\epsilon}}{\tau_{n\epsilon}} - \frac{\chi_{\epsilon n}}{\tau_{\epsilon\epsilon}} + \frac{\chi_{n\epsilon}}{\tau_{nn}} - \frac{\chi_{nn}}{\tau_{\epsilon n}} = 0$$

Conductivity is finite and the relaxation rates are not all constrained.

- Transport in WSMs $\Rightarrow$ anomalous hydrodynamics.
- *Problem 1*: negative magnetoresistance does not appear in standard order-one hydrodynamics.
- **Fix 1**: magnetic field order zero $B \sim \mathcal{O}(1) \Rightarrow \sigma \propto B^2$ for small B.
- *Problem 2*: DC conductivity infinite $\Rightarrow$ add charge relaxations, but Onsager relations are too constraining.
- **Fix 2**: use generalized relaxations, then conductivities: DC finite, Onsager reciprocal and conserve electric charge.

Thank you for the attention!

Backup Slides

Hydrodynamic frames

In hydrodynamics (like any EFT): there is freedom in writing the constitutive relations.

$$T \to T' = T + \delta T$$
$$\mu \to \mu' = \mu + \delta\mu$$
$$u^\mu \to u'^\mu = u^\mu + \delta u^\mu$$

with $\delta T, \delta\mu, \delta u^\mu \sim \mathcal{O}(\partial)$.

Observables (e.g. conductivities) must not change upon frame redefinitions.

In order-one anomalous magneto-hydrodynamics σ depends on *frame* $\Rightarrow$ not physical.

$$u^\mu \to u'^\mu = u^\mu + f(\mu, T)B^\mu$$

Two currents

$$\partial_t \delta\epsilon + \ldots = -\frac{\delta\epsilon}{\tau_{\epsilon\epsilon}} - \frac{\delta n}{\tau_{\epsilon n}} - \frac{\delta n_5}{\tau_{\epsilon n_5}}$$

$$\partial_t \delta n + \ldots = -\frac{\delta\epsilon}{\tau_{n\epsilon}} - \frac{\delta n}{\tau_{nn}} - \frac{\delta n_5}{\tau_{nn_5}}$$

$$\partial_t \delta n_5 + \ldots = -\frac{\delta\epsilon}{\tau_{n_5\epsilon}} - \frac{\delta n}{\tau_{n_5 n}} - \frac{\delta n_5}{\tau_{n_5 n_5}}$$

Onsager relations: $\tau_m \geq 0$ and arbitrary, while

$$0 = \frac{\chi_{nn_5}}{\tau_{\epsilon n_5}} + \frac{\chi_{nn}}{\tau_{\epsilon n}} - \frac{\chi_{\epsilon n_5}}{\tau_{nn_5}} + \frac{\chi_{\epsilon n}}{\tau_{\epsilon\epsilon}} - \frac{\chi_{\epsilon n}}{\tau_{nn}} - \frac{\chi_{\epsilon\epsilon}}{\tau_{n\epsilon}}$$

$$0 = \frac{\chi_{n_5 n_5}}{\tau_{\epsilon n_5}} + \frac{\chi_{nn_5}}{\tau_{\epsilon n}} - \frac{\chi_{\epsilon n_5}}{\tau_{n_5 n_5}} + \frac{\chi_{\epsilon n_5}}{\tau_{\epsilon\epsilon}} - \frac{\chi_{\epsilon n}}{\tau_{n_5 n}} - \frac{\chi_{\epsilon\epsilon}}{\tau_{n_5\epsilon}}$$

$$0 = \frac{\chi_{n_5 n_5}}{\tau_{nn_5}} - \frac{\chi_{nn_5}}{\tau_{n_5 n_5}} + \frac{\chi_{nn_5}}{\tau_{nn}} - \frac{\chi_{nn}}{\tau_{n_5 n}} + \frac{\chi_{\epsilon n_5}}{\tau_{n\epsilon}} - \frac{\chi_{\epsilon n}}{\tau_{n_5\epsilon}}$$